# Steady-state heat transfer beneath partially insulated slab-on-grade floor

#### MONCEF KRARTI

Steven Winter Associates, 50 Washington St., Norwalk, CT 06854, U.S.A.

(Received 10 June 1988 and in final form 1 November 1988)

Abstract—In this paper, the two-dimensional steady-state temperature distribution within the ground beneath a partially insulated slab-on-grade floor is presented. The solution of the governing heat conduction equation is derived using a semi-analytical technique based on a Fourier series and the Jordan—Gauss elimination method. A water table at constant temperature is assumed to exist at an arbitrary depth below the soil surface. The heat flux distribution along the floor surface as well as the total heat loss from the slab is analyzed. Some guidelines on horizontal edge insulation of slabs are given.

#### 1. INTRODUCTION

A CONSIDERABLE number of models treating the problem of heat losses from slab-on-grade floors has been developed, primarily during the last decade. These models range from detailed simulation programs to simple design methods. Sterling and Meixel [1] and Claridge [2] have produced an exhaustive survey of the models used for the calculation of heat losses in common ground-coupled structures.

However, very few authors addressed the problem of calculating heat losses from partially insulated slabon-grade floors. In fact, most of the work existing to date deals with uninsulated and to a lesser extent uniformly insulated slabs, although perimeter insulation is a common construction practice. Mitalas [3, 4] developed a finite element program that determines heat loss from a variety of basement and slab-ongrade insulation configurations, including horizontal edge insulation. However, the Mitalas method is restricted to certain insulation values and to particular geometric dimensions. Recently, Landman and Delsante [5] have used Fourier series solutions to provide detailed analytical insight regarding the effect of horizontal edge insulation on total heat losses from slabs. Their model did not allow for the existence of a water table beneath the slab.

In this paper, the steady-state temperature field distribution within the ground underneath a slab having partial insulation is analyzed. A water table is considered at a finite depth from the soil surface. The velocity of the water flow is assumed sufficiently high so as to keep a constant water temperature. To solve this problem, the procedure introduced in refs. [6–10] is employed. This procedure is called interzone temperature profile estimation (ITPE) and combines numerical and analytical techniques to arrive at the functional form of the heat conduction solution and allows the understanding of heat flux mechanisms within and at the boundaries of a conducting medium (i.e. ground).

First, the two-dimensional steady-state heat conduction solution beneath a partially slab-on-grade floor is derived. Then, the temperature distribution inside the ground is presented for various configurations. The effect of the horizontal edge insulation on the heat flux along the slab is discussed. The effectiveness of partial insulation to reduce total heat losses from the slab is also discussed.

## 2. PARTIALLY INSULATED SLAB-ON-GRADE FLOOR MODEL

Even when a slab floor is not insulated, a thermal resistance exists between room air and the slab surface. To account for this resistance, an interface conductance  $h_i$  is usually introduced and a boundary condition of the third kind is used. This boundary condition expresses the continuity of the heat flux between the lower slab surface and the interior air (at  $T_i$ ) via (1) the convective conductance  $h_0$  to the ambient air above the slab, (2) the insulation conductance  $U_i$ , (3) the slab material conductance  $U_s$ , and (4) the interface contact conductance (slab-to-earth)  $h_i$ . Therefore, the boundary condition at the slab surface can be written as

$$-k_{\rm s} \frac{\partial T}{\partial y}\bigg|_{y=0} = h[T(x,0) - T_{\rm i}] \tag{1}$$

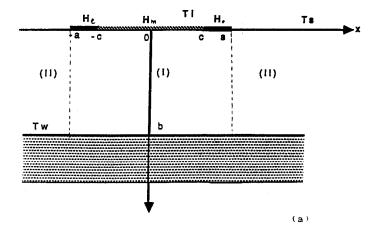
where  $k_s$  is the thermal conductivity of the soil (assumed isotropic),  $T_i$  the air temperature above the slab (i.e. the building interior temperature), and h the equivalent air-insulation-slab-soil conductance, given by

$$h = (h_0^{-1} + U_i^{-1} + U_s^{-1} + h_i^{-1})^{-1}.$$
 (2)

Note that in this paper, the insulation conductance  $U_i$  is not necessarily uniform along the slab-on-grade floor.

Figure 1 shows the vertical cross-section of a house with slab-on-grade floor partially insulated. Note that

NOMENCLATURE			
a	half width of ground-coupled	$\Delta T_{0}$	dimensionless temperature
	building [m]	$T_{ m i}$	building air temperature [K]
b	water table depth [m]	$T_{\mathrm{s}}$	soil surface temperature [K]
$C_n$	general term in a Fourier series	$T_{\mathbf{w}}$	water table temperature [K]
	expansion	$U_{\mathrm{o}}$	envelope material conductance
c	basement depth [m]		$[W m^{-2} K^{-1}]$
f	functions of one of the space	$U_{i}$	insulation layer conductance
	coordinates [K]		$[W m^{-2} K^{-1}]$
$f_n$	Fourier coefficients	x, y	space coordinates [m].
H	ratio, $h/k_s$ [m <sup>-1</sup> ]		
h	overall heat transfer conductance	Greek symbols	
	$[W m^{-2} K^{-1}]$	$\alpha, \beta, \gamma$	_ •
h <sub>i</sub> h <sub>o</sub>	inside surface convective heat transfer		and (27)
	coefficient [W m <sup>-2</sup> K <sup>-1</sup> ]	$\alpha', \beta', \gamma'$	coefficients defined in equation (29)
	outside surface convective heat		, $\mu_n$ eigenvalues [m <sup>-1</sup> ].
	transfer coefficient [W m <sup>-2</sup> K <sup>-1</sup> ]		
$k_{\rm s}$	soil thermal conductivity		
	$[W m^{-1} K^{-1}]$	Subscripts	
L	distance from building center to a	e	edge of the slab
	boundary where soil temperature is	m	middle of the slab
	undisturbed [m]	I	zone (I)
T	temperature [K]	II	zone (II).



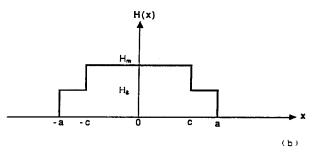


Fig. 1. (a) Slab-on-grade floor with non-uniform horizontal insulation; (b) ratio,  $H = h/k_s$ , distribution showing reduced conductance beyond slab edges (x = c). Insulation length is (c - a).

a typical slab-on-grade floor foundation includes vertical foundation walls and footings in addition to the horizontal slab and thermal insulation layer. However, in most cases, the thermal conductivity of the ground is similar to that of the foundation walls and footings (usually made of concrete). Therefore, thermally these elements of the floor foundation can be considered as integral parts of the ground medium, and the model of Fig. 1 can be applied.

The steady-state temperature distribution T(x, y) inside the ground for a partially insulated slab-ongrade floor configuration as shown in Fig. 1, can be determined by solving the following equation:

$$\Delta T = 0 \tag{3}$$

with

$$T = T_{\rm w}$$
 for  $y = b$ 

$$T = T_s$$
 for  $y = 0$  and  $|x| > a$ 

$$\frac{\partial T}{\partial v} = H(x)(T - T_i)$$
 for  $y = 0$  and  $|x| < a$ 

where H(x) is the ratio of the equivalent air-insulation-slab-soil conductance to the soil thermal conductivity (i.e.  $H(x) = h(x)/k_s$ ). In this paper, H(x) is defined by

$$H(x) = H_{m} \quad \text{if } |x| < c$$

$$H(x) = H_{e} \quad \text{if } c < |x| < a.$$

In Fig. 1(b), the variation of the function H(x) is shown. Note that c measures how far the insulation is placed from the slab center and that a-c is the width of this edge insulation.

Figure 1(a) shows that the surfaces, x = -a and a, divide the ground medium into three zones. Because of the symmetry around the axis x = 0 the temperature T(x, y) will be determined only in zones (I) and (II). Let f(y) be the temperature profile along the surface x = a; then the solution of equation (1) in zone (I) is

$$T_{1}(x, y) = \frac{2}{b} \sum_{n=1}^{\infty} \frac{\sin \nu_{n} y}{\nu_{n}} \{ [T_{s} - (-1)^{n} T_{w}] \times [1 - e^{-\nu_{n}(x-a)}] + \nu_{n} f_{n} e^{-\nu_{n}(x-a)} \}$$
 (4)

while in zone (II), the temperature  $T_{II}(x, y)$  is given by

$$T_{11}(x, y) = \frac{2}{b} \sum_{n=1}^{\infty} f_n \sin \nu_n y \frac{\cosh \nu_n x}{\cosh \nu_n a}$$

$$+ \frac{2}{a} T_w \sum_{n=1}^{\infty} \frac{(-1)^n}{\mu_n} \cos \mu_n x \frac{\sinh \mu_n y}{\sinh \mu_n b}$$

$$+ \frac{2}{a} \sum_{n=1}^{\infty} C_n \cos \mu_n x \frac{\sinh \mu_n (b-y)}{\sinh \mu_n b}$$
 (5)

where

$$v_n = \frac{n\pi}{b}; \quad f_n = \int_0^b f(y) \sin v_n y \, dy;$$
$$\mu_n = \frac{(2n-1)\pi}{2a}$$

and  $C_n$ , like  $f_n$ , are Fourier coefficients to be determined.

The continuity of the heat flux at the surface x = a, gives the condition

$$\left. \frac{\partial T_{\mathbf{i}}}{\partial x} \right|_{\mathbf{x} = a} = \left. \frac{\partial T_{\mathbf{i}}}{\partial x} \right|_{\mathbf{x} = a} \tag{6}$$

or

$$\frac{2}{b} \sum_{n=1}^{\infty} \sin \nu_n y \{ \nu_n f_n - [T_s - (-1)^n T_w] \}$$

$$= \frac{-2}{b} \sum_{n=1}^{\infty} \nu_n \tanh \nu_n a f_n \sin \nu_n y$$

$$+\frac{2}{a}T_{w}\sum_{n=1}^{\infty}\frac{\sinh \mu_{n}y}{\sinh \mu_{n}b}+\frac{2}{a}\sum_{n=1}^{\infty}(-1)^{n}\mu_{n}C_{n}\frac{\sinh \mu_{n}(b-y)}{\sinh \mu_{n}b}.$$

Multiplying the above equality by  $\sin v_p y$  (p = 1, 2, ...), and integrating the resultant equation over [0, b] yields

$$-\nu_{p}f_{p} + [T_{s} - (-1)^{p}T_{w}] = \nu_{p} \tanh \nu_{p}af_{p}$$

$$+ (-1)^{p} \tanh \nu_{p}aT_{w} - \frac{2}{a} \sum_{n=1}^{\infty} (-1)^{n}C_{n} \frac{\nu_{p}\mu_{n}}{\mu_{n}^{2} + \nu_{p}^{2}}.$$

After rearrangement, this expression can be put in the form

$$f_p = \alpha_p + \sum_{n=1}^{\infty} \beta_{n,p} C_n \tag{7}$$

where

$$\alpha_p = \frac{1}{\nu_p (1 + \tanh \nu_p a)} \left\{ [T_s - (-1)^p T_w] - (-1)^p \tanh \nu_n a T_w \right\}$$
 (8)

and

$$\beta_{n,p} = -\frac{2(-1)^n v_m}{a(1 + \tanh v_n a)(\mu_n^2 + v_n^2)}.$$
 (9)

From the third-kind boundary condition of equation (3) at y = 0, coefficients  $C_n$  are subject to

$$\left. \frac{\partial T_{II}}{\partial y} \right|_{y=0} = H(x)(T_{II} - T_{i})$$

or, using expression (5)

$$\frac{2}{b} \sum_{n=1}^{\infty} v_n f_n \frac{\cosh v_n x}{\cosh v_n a} + \frac{2}{a} T_w \sum_{n=1}^{\infty} \frac{(-1)^n}{\sinh \mu_n b} \cos \mu_n x$$
$$- \frac{2}{a} \sum_{n=1}^{\infty} \mu_n C_n \coth \mu_n b \cos \mu_n x$$
$$= H(x) \left( \frac{2}{a} \sum_{n=1}^{\infty} C_n \cos \mu_n x - T_i \right). \quad (10)$$

Multiplying this equation by  $\cos \mu_p x$  and integrating over the interval [-a, a] gives after some arrangement

$$C_p = \alpha'_p + \sum_{n=1}^{\infty} \beta'_{n,p} f_p + \sum_{n=1}^{\infty} \gamma'_{n,p} C_n$$
 (11)

964 M. Krarti

with

$$\alpha'_{p} = \frac{(H_{\rm m} - H_{\rm e})\sin\mu_{p}c - (-1)^{p}H_{\rm m}}{\mu_{p}(H_{\rm e} + \mu_{p}\coth\mu_{p}b)}T_{\rm i} + \frac{(-1)^{p}}{\sinh\mu_{p}b}T_{\rm w}$$
$$\beta'_{n,p} = \frac{2}{b}\frac{(-1)^{p}\nu_{n}\mu_{p}}{(\mu_{p}^{2} + \nu_{n}^{2})(H_{\rm e} + \mu_{p}\coth\mu_{p}b)}$$

and

$$\begin{split} \gamma_{n,p}' &= \frac{2}{a} \frac{(H_{\rm e} - H_{\rm m})}{(H_{\rm e} + \mu_p \coth \mu_p b)} \\ &\qquad \times \left\{ \frac{\sin{(\mu_n - \mu_p)c}}{(\mu_n - \mu_p)} + \frac{\sin{(\mu_n + \mu_p)c}}{(\mu_n + \mu_p)} \right\}. \end{split}$$

Note that the case of a uniformly insulated slab is obtained when  $H_{\rm m} = H_{\rm e}$ . Coefficients  $\alpha'_{p}$  and  $\gamma'_{n,p}$  are then reduced to zero and coefficients  $C_p$ 's are solely a function of  $f_p$ . Reference [7] treats this particular case in detail. In the general case (i.e.  $H_m \neq H_e$ ), the system of equations (7) and (11) can be solved for the  $f_p$ 's and  $C_p$ 's numerically as follows: the sum in both equations (7) and (11) is first truncated to a finite number of terms, N. By varying the value of p from 1 to N in equations (7) and (11), a linear system of 2N equations with 2N unknowns (the coefficients  $f_p$ and  $C_p$ , p = 1, 2, ..., N) is obtained. This system can be solved using the Gauss-Jordan elimination method. Once coefficients  $f_p$  and  $C_p$  are found, they are substituted in equations (5) and (6) (in which the sums are also truncated to N terms) to obtain the temperature distributions  $T_{I}(x, y)$  and  $T_{II}(x, y)$ , respectively. For the slab configurations considered in this paper, it was found that N = 50 gives accurate estimations. Addition of other terms does not alter the results for  $T_1$  and  $T_{11}$  significantly (less than a 0.01 K variation in soil temperature, for the cases treated in this paper).

#### 3. SOIL TEMPERATURE DISTRIBUTION

Figure 2 shows the temperature distribution within soil when a slab of width 2a = 10 m is present. The temperature of air above the slab is  $T_i = 21^{\circ}\text{C}$ , while the soil surface temperature is  $T_s = 16^{\circ}\text{C}$ . A water table at a depth b = 5 m below the surface has a constant temperature  $T_w = 11^{\circ}\text{C}$ . Six different insulation configurations are considered in Fig. 2.

- (a) Uniformly perfectly conducting slab  $(H_m = H_e = \infty)$ , Fig. 2(a).
- (b) Uninsulated slab with a horizontal edge insulation extending 1 m and having zero thermal conductivity ( $H_m = \infty$ ,  $H_e = 0$ , and c = 4 m), Fig. 2(b).
- (c) Nominally uniformly insulated slab ( $H_{\rm m} = H_{\rm e} = 1.0~{\rm m}^{-1}$ ), Fig. 2(c).
- (d) Uniformly well-insulated slab  $(H_m = H_e = 0.2 \text{ m}^{-1})$ , Fig. 2(d).

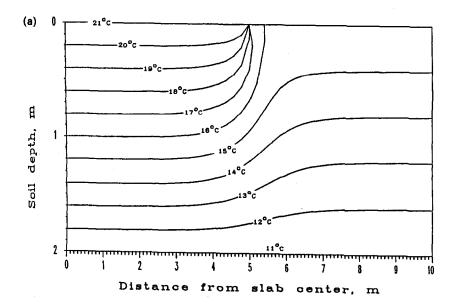
(e) Partially insulated slab with edge insulation  $(H_m = 1.0 \text{ m}^{-1}, H_e = 0.2 \text{ m}^{-1}, \text{ and } c = 4 \text{ m})$ , Fig. 2(e).

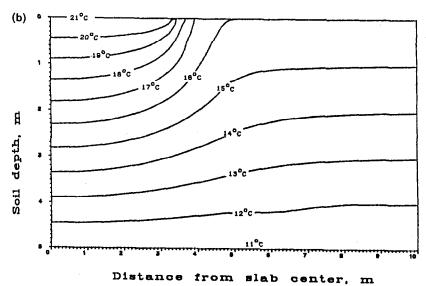
(f) Partially insulated slab with well-insulated center ( $H_{\rm m}=0.2~{\rm m}^{-1}$ ,  $H_{\rm e}=1.0~{\rm m}^{-1}$ , and c=4 m), Fig. 2(f).

In the case of an uninsulated slab (Fig. 2(a)), the edge is a singularity point, that is the temperature at this point takes all the values from  $T_i = 21^{\circ}$ C to  $T_s = 16$ °C. Therefore, the heat flux at the edge of an uninsulated slab is very important (mathematically, it is infinity). To reduce heat losses from slabs, it is then preferable to use edge insulation. Figure 2(b) shows some isotherms underneath an uninsulated slab with a perfect horizontal edge insulation (an idealized insulation material with zero thermal conductivity) extending (a-c) = 2 m from the slab edge. The singularity point is removed from the edge and the temperature along the insulated area (region of slab surface between x = 3 and 5 m) decreases from  $T_i = 21$ °C to  $T_s = 16^{\circ}$ C. It is interesting to note by comparing Figs. 2(a) and (b) that when an edge insulation is added, the isotherm  $T = 20^{\circ}$ C moves closer to the slab surface (especially near x = 4 m). This implies that heat losses increase in the central zone of the slab, as insulation is added to its perimeter.

In reality and even in uninsulated slabs, the conductance coefficient H is a finite number. For a 0.10 m (4 in.) concrete slab,  $H = 4 \text{ m}^{-1}$  if soil thermal conductivity is  $k_s = 1 \text{ W m}^{-2} \text{ K}^{-1}$ . When an insulation of R-4.3 (R=0.75 m<sup>2</sup> K W<sup>-1</sup>) is uniformly added to the slab surface, H becomes 1 m<sup>-1</sup>. With an insulation of R-27 ( $R = 4.75 \text{ m}^2 \text{ K W}^{-1}$ ), H is 0.2 m<sup>-1</sup>. Figures 2(c) and (d) show representative isotherms for uniformly insulated slabs with  $H_m =$  $H_e = 1$  and 0.2 m<sup>-1</sup>, respectively. In the case of Fig. 2(c) the temperature along the slab decreases from T = 20.5°C at the center to  $T_s = 16$ °C at the edge. Heat flux is still important around the perimeter of the slab, since a temperature change of 2.5 K occurs between x = 4 and 5 m. Figure 2(d) is the very particular case when the slab surface temperature becomes uniform and equal to the adjacent soil surface temperature  $T_s = 16^{\circ}$ C. In this case heat flow is unidimensional. More discussion of this case and a uniformly insulated slab can be found in ref. [7].

Figures 2(e) and (f) represent the isotherms in soil when insulation is not uniform. The common case where insulation is added to the slab perimeter is shown in Fig. 2(e). The isotherm patterns in this configuration are very similar to those encountered in the ideal case of Fig. 2(b). However, the temperature at the slab surface is a little lower and varies from  $20^{\circ}$ C at the center to  $17^{\circ}$ C at the edge. An unusual case, where insulation is placed at the middle of the slab so that  $H_{\rm m}$  is larger than  $H_{\rm e}$ , is presented in Fig. 2(f). Besides the noticeable change in the temperature distribution within the soil, the slab surface temperature now increases from about  $17.2^{\circ}$ C at the center to





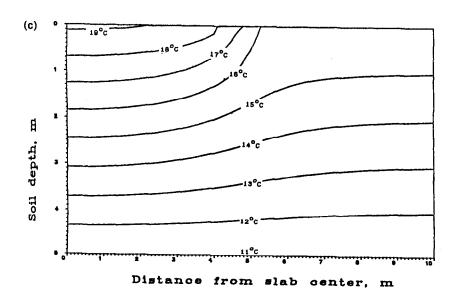
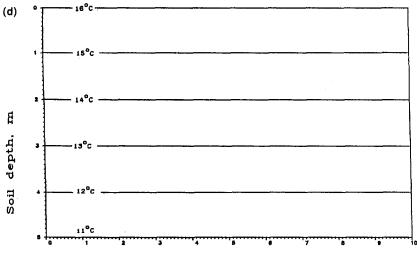


Fig. 2. Earth temperature isotherms beneath a slab-on-grade floor with water table depth  $b=5~{\rm m}$ : (a)  $H_{\rm m}=H_{\rm e}=\infty$ ; (b)  $H_{\rm m}=\infty$ ,  $H_{\rm e}=0~{\rm m}^{-1}$ ,  $c=4~{\rm m}$ ; (c)  $H_{\rm m}=H_{\rm e}=1~{\rm m}^{-1}$ ; (d)  $H_{\rm m}=H_{\rm e}=0.2~{\rm m}^{-1}$ ; (e)  $H_{\rm m}=1.0~{\rm m}^{-1}$ ,  $H_{\rm e}=0.2~{\rm m}^{-1}$ ,  $t=1.0~{\rm m}^{$ 



Distance from slab center, m

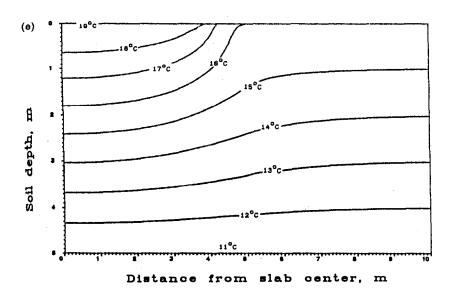


Fig. 2—Continued.

Distance from slab center, m

18.5°C at x = 4.5 m and decreases to 16°C at the edge (i.e. x = 5 m).

#### 4. HEAT FLUX DISTRIBUTION

The steady-state heat flux distribution q(x), along the slab surface is determined from the third-kind boundary condition of equation (3)

$$q(x) = k_s \frac{\partial T}{\partial y} \bigg|_{y=0} = k_s H(x) (T_{II} - T_i)$$
 (12)

or after some rearrangement

$$q(x) = -\frac{2}{a}k_{s}H(x)\sum_{n=1}^{\infty} \left(C_{n} + \frac{(-1)^{n}}{\mu_{n}}T_{i}\right)\cos\mu_{n}x.$$
(13)

Figure 3 shows the heat flux distribution along a slab of width 2a = 10 m. The temperature above the slab is  $T_i = 21^{\circ}$ C. The soil surface temperature is at  $T_s = 16$ °C, while a water table at a depth of about b = 5 m exists and is at the temperature  $T_w = 11$ °C. Two insulation configurations are presented in Fig. 3. In the first configuration  $H_{\rm m} = H_{\rm e} = 1~{\rm m}^{-1}$  (case without edge insulation) and in the second,  $H_{\rm m}=1$  $m^{-1}$  and  $H_e = 0.2 m^{-1}$  (case with edge insulation). Figure 3 indicates clearly that by adding perimeter insulation, heat losses are reduced from the slab edge as expected. However, there is a slightly greater heat loss from the slab center especially from the portion of the slab located between x = 3 and 4 m. This phenomenon occurs because the added edge insulation increases the temperature in the slab surface near x = 4 m so that more heat is lost from this area. Note that the overall effect of the perimeter insulation is a net reduction of the total heat losses from the slab surface as will be discussed in the next section.

#### 5. TOTAL HEAT LOSSES FROM SLAB SURFACE

The total heat losses Q from the slab floor is calculated by integrating the heat flux function q(x), over the interval [-a, a]

$$Q = \int_{-a}^{a} q(x) dx = 2 \int_{0}^{c} H_{m}(T_{I}(x) - T_{i}) dx + 2 \int_{c}^{a} H_{e}(T_{II} - T_{i}) dx$$
 (14)

or, after integration and simplification

$$Q = \frac{4}{a}k_{s} \sum_{n=1}^{\infty} \left( (H_{m} - H_{c}) \frac{\sin \mu_{n} c}{\mu_{n}} - \frac{(-1)^{n}}{\mu_{n}} \right) \left[ C_{n} + \frac{(-1)^{n}}{m u_{n}} T_{i} \right]. \quad (15)$$

Figures 4 and 5 show the total heat losses Q as a function of the insulation location c, for  $H_{\rm m}=4~{\rm m}^{-1}$  (poorly insulated) and  $H_{\rm m}=1~{\rm m}^{-1}$  (moderately insulated). In each figure, various edge insulation values are considered ( $H_{\rm e}=H_{\rm m}$ ;  $H_{\rm e}=0.8$ ,  $H_{\rm e}=0.6$ ,  $H_{\rm e}=0.4$ ,  $H_{\rm e}=0.2$ ,  $H_{\rm e}=0$ ). Both figures indicate that the increase in edge insulation (either through increasing the  $H_{\rm e}$  value or by decreasing the value of c) reduces the total heat losses from the slab floor. Note that the added insulation is subject to the law of diminishing returns. For example and in the poorly insulated case with c=2 m, a decrease of  $H_{\rm e}$  from 0.8 to 0.6 m<sup>-1</sup> reduces heat losses by 2 W m<sup>-2</sup> or 4.3%. However, a same decrease of  $H_{\rm e}$  from 0.4 to 0.2 m<sup>-1</sup> implies a reduction of heat losses by 3 W m<sup>-2</sup> or 16%.

In particular for a poorly insulated slab (Fig. 4), most of the reduction in heat losses is achieved for an edge insulation width of 0.5 m (i.e. c = 4.5 m). In fact,

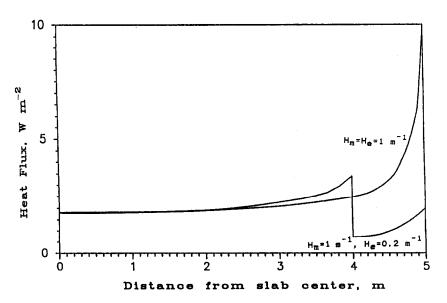


Fig. 3. Effect of edge insulation on local heat flux distribution along an insulated slab-on-grade floor with  $T_i = 21^{\circ}\text{C}$ ,  $T_s = 16^{\circ}\text{C}$ ,  $T_w = 11^{\circ}\text{C}$ , c = 4 m.

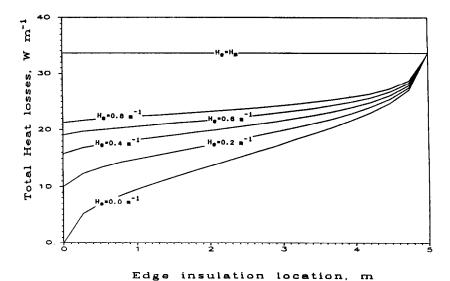


Fig. 4. Effect of edge insulation and its width on the total heat losses from a poorly insulated slab  $(H_m = 4 \text{ m}^{-1})$ .

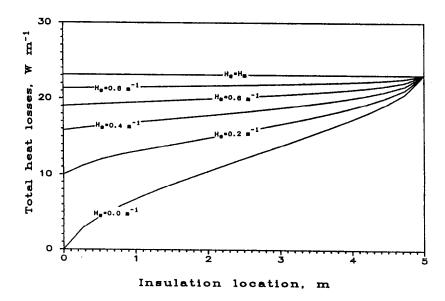


Fig. 5. Effect of edge insulation and its width on the total heat losses from a well insulated slab  $(H_{\rm m}=1~{\rm m}^{-1})$ .

beyond this value, the heat loss curves are fairly flat for non-perfect edge insulation (i.e.  $H_e \neq 0$ ).

#### 6. CONCLUSIONS

The steady-state temperature distribution beneath both a totally and partially insulated slab-on-grade floor is determined using a semi-analytical procedure. The dependence of heat transfer from the slab surface upon the edge insulation width and resistance is analyzed in particular. It is shown in particular that the edge insulation follows the law of diminishing returns and that most heat loss reduction is provided by the first  $0.5 \,\mathrm{m}$  of the insulation. Finally, it is important to mention that two variables (thermally almost equivalent) are to be considered in designing slab edge insulation: insulation width and U-value. Economics should determine whether to use wider or more resistant insulations around the perimeter of the slab-ongrade floor.

#### REFERENCES

 R. L. Sterling and G. D. Meixel, Review of underground heat transfer research, Earth Sheltered Performance and Evaluation Proceedings, Second Technical Conference (Edited by L. L. Boyer), pp. 67-74. Oklahoma State University (1981).

- D. E. Claridge, Design methods for earth-contact heat transfer. In *Progress in Solar Energy* (Edited by K. Boer). American Solar Energy Society, Boulder, Colorado (1987).
- 3. G. P. Mitalas, Calculation of below-grade residential heat loss; low-rise residential building, ASHRAE Trans. 93(1), 743-783 (1987).
- G. P. Mitalas, Calculation of basement heat loss, ASHRAE Trans. 80(1B), 420-438 (1983).
- K. A. Landman and A. E. Delsante, Steady-state heat losses from a building floor slab with horizontal edge insulation, *Bldg Environ. J.* 22, 57-60 (1987).
- M. Krarti, Developments in ground-coupling heat transfer, Ph.D. Dissertation, University of Colorado, Boulder, Colorado (1987).

- M. Krarti, D. E. Claridge and J. F. Kreider, The ITPE technique applied to steady-state ground-coupling problems, Int. J. Heat Mass Transfer 31, 1885–1898 (1988).
- 8. M. Krarti, D. E. Claridge and J. F. Kreider, ITPE technique applications to time-varying two-dimensional ground-coupling problems, *Int. J. Heat Mass Transfer* 31, 1899-1911 (1988).
- M. Krarti, D. E. Claridge and J. F. Kreider, The ITPE method applied to time-varying three-dimensional ground-coupling problems, *Proc. Tenth Annual ASME* Solar Energy Conf., pp. 453-460 (April 1988).
- M. Krarti and D. E. Claridge, Two-dimensional heat transfer from earth-sheltered buildings, to be presented at the 11th Annual ASME Solar Energy Conf. (2-5 April 1989).

## TRANSFERT DE CHALEUR PERMANENT SOUS UNE DALLE PARTIELLEMENT ISOLEE

Résumé—On présente la distribution bidimensionnelle et stationnaire de température dans le sol sous une dalle partiellement isolée. La solution de l'équation de conduction thermique est obtenue par une technique semi-analytique basée sur les séries de Fourier et la méthode d'élimination de Jordan—Gauss. Une nappe d'eau à température constante est située à une profondeur arbitraire sous la surface du sol. On analyse la distribution du flux de chaleur sur la surface de la dalle ainsi que la perte de chaleur totale. On donne les lignes directrices sur l'isolation de la face horizontale des dalles.

## STATIONÄRER WÄRMEDURCHGANG UNTERHALB EINES TEILWEISE ISOLIERTEN FUSSBODENS

Zusammenfassung—In dieser Arbeit wird die zweidimensionale Temperaturverteilung in der Erde unterhalb eines teilweise isolierten Fußbodens für den stationären Zustand dargestellt. Die Wärmeleitungsgleichung wurde mit einem halb-analytischen Verfahren gelöst, welches auf Fourier-Reihen und dem Gauss-Jordan-Eliminationsverfahren aufbaut. In einer beliebigen Tiefe wurde Grundwasser mit konstanter Temperatur angenommen. Die Wärmeverluste und deren Verteilung entlang der Fußbodenfläche wurden bestimmt. Empfehlungen für die Wärmedämmung an den Fußboden-Eckpunkten werden ausgesprochen.

## СТАЦИОНАРНЫЙ ТЕПЛОПЕРЕНОС ПОД ЧАСТИЧНО ИЗОЛИРОВАННЫМ НАСТИЛОМ ИЗ ПЛИТ, РАСПОЛОЖЕННОМ НА УРОВНЕ ЗЕМЛИ

Аниотация—Представлено двумерное стационарное распределение температуры в грунте под частично изолированным настилом из плит, расположенном на уровне земли. Решение уравнения теплопроводности получено с помощью полуэмпирического подхода, основанного на разложении в ряд Фурье и методе исключения Жордана-Гаусса. Предполагается, что на определенной глубине в грунте находится водное зеркало при постоянной температуре. Анализируются распределение теплового потока по поверхности настила, а также суммарные потери тепла плитой. Дан ряд рекомендаций относительно изоляции горизонтального края плит.